Fin Flutter Analysis for TRA Level 3 Certification Rocket
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From Peak of Flight Newsletter 411:

Fin Parameters:
cr = root chord length
ct = tip chord length
b = fin height
t = fin thickness
G = fin material shear modulus

Atmospheric Parameters:
Cs0 = speed of sound at sea level
P0 = atmospheric pressure at sea level
P(h) = pressure as a function of altitude, h
Cs(h) = speed of sound as a function of altitude, h

H = atmospheric scale factor, gas and temperature
M = mean molecular mass of one atmospheric particle

$cr := 12.00\ in$
$cr = 304.8\ mm$

$ct := 7\ in$
$ct = 177.8\ mm$

$b := 8.00\ in$
$b = 203.2\ mm$

$t := .125\ in$
$t = 3.175\ mm$

$G := 16.5\ GPa$

$k = \left(1.381 \cdot 10^{-23}\right) \frac{kg \cdot m^2}{s^2 \cdot K}$

$R = 8.314 \frac{kg \cdot m^2}{s^2 \cdot K \cdot mol}$

$M := 0.029 \frac{kg}{mol}$

$H(T) := \frac{R \cdot T}{M \cdot g}$

$$T := \begin{bmatrix} 11 \\ 7.1 \\ 3.1 \\ -0.8 \\ -4.8 \end{bmatrix} \ °C$$

$$H(T) = \begin{bmatrix} 8.424 \cdot 10^3 \\ 8.307 \cdot 10^3 \\ 8.193 \cdot 10^3 \\ 8.076 \cdot 10^3 \\ 7.962 \cdot 10^3 \\ 7.845 \cdot 10^3 \end{bmatrix} \ m$$

$P_0 := 1 \ atm$

$P_0 = (1.013 \cdot 10^5) \ Pa$

$$P(h) := P_0 \cdot \exp\left(\frac{-h}{H(T)}\right)$$

$$h := \begin{bmatrix} 0 \\ 2000 \\ 4000 \\ 6000 \\ 8000 \\ 10000 \end{bmatrix} \ ft$$

$$h = \begin{bmatrix} 0 \\ 609.6 \\ 1.219 \cdot 10^3 \\ 1.829 \cdot 10^3 \\ 2.438 \cdot 10^3 \\ 3.048 \cdot 10^3 \end{bmatrix} \ m$$

$$P(h) := P_0 \cdot \exp\left(\frac{-h}{H(T)}\right)$$

$$P(h) = \begin{bmatrix} 1.013 \cdot 10^5 \\ 9.416 \cdot 10^4 \\ 8.732 \cdot 10^4 \\ 8.079 \cdot 10^4 \\ 7.46 \cdot 10^4 \\ 6.871 \cdot 10^4 \end{bmatrix} \ Pa$$

$C_{s0} := 344 \ \frac{m}{}$

s

$$C_s(h) := C_{s0} \cdot \exp\left(-.1 \cdot \left(\frac{h}{H(T)}\right)\right)$$

$$C_s(h) = \begin{bmatrix} 344 \\ 341.485 \\ 338.919 \\ 336.298 \\ 333.625 \\ 330.892 \end{bmatrix} \frac{m}{s}$$

Derived Parameters

$$\lambda := \frac{ct}{cr}$$

$$S := \frac{1}{2}(cr + ct) \cdot b$$

$$B := \frac{b^2}{S}$$

$$NT := \frac{t}{cr}$$

$$V_f := 1.223 \cdot C_{s0} \cdot \exp\left(0.4 \cdot \frac{h}{H(T)}\right) \cdot \sqrt{\frac{G}{P_0}} \cdot \sqrt{\frac{(2+B)}{(1+\lambda)}} \cdot \left(\frac{NT}{B}\right)^{\frac{3}{2}}$$

$$V_f = \begin{bmatrix} 312.929 \\ 322.25 \\ 332.121 \\ 342.596 \\ 353.708 \\ 365.541 \end{bmatrix} \frac{m}{s}$$

$$V_f = \begin{bmatrix} 700.003 \\ 720.854 \\ 742.933 \\ 766.365 \\ 791.223 \\ 817.692 \end{bmatrix} mph$$